1. $(x-2)^2 + y^2 = 9$

circle

center: $(2,0)$ radius: 3

see graph paper

2. Endpoints of diameter: $C(2,-5)$ $D(6,1)$

center: $\left(\frac{2+6}{2}, \frac{-5+1}{2}\right) = (4,-2)$

radius = distance between $(2,-5)$ & $(4,-2)$

$r = \sqrt{(4-2)^2 + (-2-(-5))^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$

$(x-h)^2 + (y-k)^2 = r^2$

$\boxed{(x-4)^2 + (y+2)^2 = 13}$

3. $x^2 + y^2 - 10x + 6y - 2 = 0$

$(x^2 - 10x \quad) + (y^2 + 6y \quad) = 2$

$(x^2 - 10x + 25) + (y^2 + 6y + 9) = 2 + 25 + 9$

$(x-5)^2 + (y+3)^2 = 36$

$\boxed{\text{center: } (5,-3) \quad \text{radius: } 6}$

$\frac{-10}{2} = -5$

$(-5)^2 = 25$

$\frac{6}{2} = 3 \quad 3^2 = 9$

4. parabola focus $(1,-1)$ directrix $y=3$

opens down $(x-h)^2 = -4a(y-k)$

axis of symmetry: $x = 1$

y-coord of vertex: halfway between -1 & 3

vertex: $(1,1)$

a = distance from focus to vertex = 2

$(x-1)^2 = -4(2)(y-1)$

$\boxed{(x-1)^2 = -8(y-1)}$

5. $(y-4)^2 = 8(x+2)$ opens right

$4a = 8$ $a = 2$

$\boxed{\text{vertex: } (-2,4)}$

$\boxed{\text{focus: } (0,4)}$

$\boxed{\text{directrix: } x = -4}$

see graph paper

6. $\dfrac{9x^2+4y^2}{36}=\dfrac{36}{36}$ $\qquad \dfrac{x^2}{4}+\dfrac{y^2}{9}=1$

ellipse
center: $(0,0)$
foci: $(0,-\sqrt{5})$ & $(0,\sqrt{5})$

$a^2=9 \qquad a=3 \updownarrow$ (under y)

$b^2=4 \qquad b=2 \leftrightarrow$ (under x)

$c^2=a^2-b^2=9-4=5$

$c=\sqrt{5}\approx 2.2$

see graph paper

7. $\dfrac{(x+1)^2}{4}+\dfrac{(y-2)^2}{9}=1$

$a^2=9 \qquad a=3 \updownarrow$

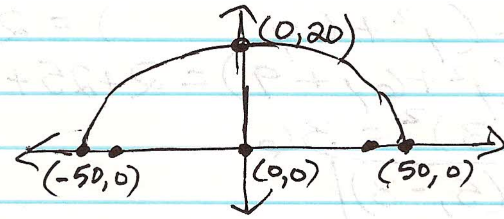
$b^2=4 \qquad b=2 \leftrightarrow$

$c^2=a^2-b^2=9-4=5$

$c=\sqrt{5}$

center: $(-1,2)$ vertices: $(-1,5)$ & $(-1,-1)$ foci: $(-1,2+\sqrt{5})$ & $(-1,2-\sqrt{5})$

8.



a = dist. from center to vertex = 50

b = dist. from center to covertex = 20

$c^2=a^2-b^2$

$c^2=50^2-20^2=2100$

$c=\sqrt{2100}=45.8$

foci are 45.8 ft from the center.

9. $\dfrac{16y^2-9x^2}{144}=\dfrac{144}{144}$

$\dfrac{y^2}{9}-\dfrac{x^2}{16}=1$

$a^2=9 \qquad a=3 \updownarrow$ (under y)

$b^2=16 \qquad b=4 \leftrightarrow$ (under x)

$c^2=a^2+b^2=9+16=25 \qquad c=5$

asymptotes: $m=\pm 3/4$

$\boxed{y=\pm 3/4\, x}$

center: $(0,0)$ vertices: $(0,-3)$ & $(0,3)$ foci: $(0,-5)$ & $(0,5)$

see graph paper

10. $\dfrac{(y+2)^2}{4} - \dfrac{(x-3)^2}{9} = 1$

$a^2 = 4 \quad a = 2 \updownarrow$ (under y)

$b^2 = 9 \quad b = 3 \leftrightarrow$ (under x)

$c^2 = a^2 + b^2 = 4 + 9 = 13$

$c = \sqrt{13}$

center: $(3,-2)$
transverse axis: $x = 3$
vertices: $(3,0)$ & $(3,-4)$
foci: $(3,-2-\sqrt{13})$ & $(3,-2+\sqrt{13})$
asymptotes: $y+2 = \pm \frac{2}{3}(x-3)$

11. center $(2,3)$, focus $(0,3)$, vertex $(1,3)$

$a = 1$ (dist. from center to vertex)

$c = 2$ (dist. from center to focus)

$a^2 + b^2 = c^2$

$1^2 + b^2 = 2^2$

$b^2 = 2^2 - 1^2 = 3$

$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1 \qquad \dfrac{(x-2)^2}{1} - \dfrac{(y-3)^2}{3} = 1$

$$\boxed{(x-2)^2 - \frac{(y-3)^2}{3} = 1}$$

12. $\begin{cases} 4x - 3y + z = -2 \\ 5y - z = 6 \\ 2x + 8z = -9 \end{cases}$ $\quad \boxed{\left[\begin{array}{ccc|c} 4 & -3 & 1 & -2 \\ 0 & 5 & -1 & 6 \\ 2 & 0 & 8 & -9 \end{array}\right]}$

13. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\quad \begin{cases} x = -2 \\ y = 4 \\ z = 1 \end{cases}$

consistent
one solution: $(-2,4,1)$

14. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right]$ $\quad \begin{cases} x = -2 \\ y = 4 \\ 0 = 1 \end{cases}$

inconsistent
no solution

15. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\quad \begin{cases} x = -2 \\ y + 3z = 4 \\ 0 = 0 \end{cases}$

consistent
infinite # of solutions

solution set: $\{(x,y,z) \mid x = -2,\ y = -3z + 4,$ & z is any real #$\}$

16. $\left[\begin{array}{ccc|c} 2 & 4 & 5 & -2 \\ 1 & 2 & 3 & 4 \\ 3 & 3 & 7 & 1 \end{array}\right]$ $R_2 = -3r_2 + r_3$

$-3(1)+3=0$
$-3(2)+3=-3$
$-3(3)+7=-2$
$-3(4)+1=-11$

$$\boxed{\left[\begin{array}{ccc|c} 2 & 4 & 5 & -2 \\ 0 & -3 & -2 & -11 \\ 3 & 3 & 7 & 1 \end{array}\right]}$$

17. $\begin{vmatrix} 2 & -5 \\ 4 & x \end{vmatrix} = 6$

$2(x) - (-5)(4) = 6$

$2x + 20 = 6$

$2x = -14$

$\boxed{x = -7}$

18. $\begin{vmatrix} -1 & 2 & 1 \\ 2 & -2 & 3 \\ 3 & -1 & 0 \end{vmatrix}$

Expansion by minors:

$-1\begin{vmatrix} -2 & 3 \\ -1 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 3 & 0 \end{vmatrix} + 1\begin{vmatrix} 2 & -2 \\ 3 & -1 \end{vmatrix} =$

$-1(0-(-3)) - 2(0-9) + 1(-2-(-6)) =$

$-1(3) - 2(-9) + 1(4) = -3 + 18 + 4 = \boxed{19}$

Diagonals: $-6 \quad 3 \quad 0 = -3$

$$\left|\begin{array}{ccc|cc} -1 & 2 & 1 & -1 & 2 \\ 2 & -2 & 3 & 2 & -2 \\ 3 & -1 & 0 & 3 & -1 \end{array}\right|$$

$0 \quad 18 \quad -2 = 16$

$16 - (-3) = \boxed{19}$

19. $\begin{cases} 2x - y + z = 3 \\ x - y - z = 4 \\ x + 2y - 2z = 1 \end{cases}$ $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 1 & -1 & -1 & 4 \\ 1 & 2 & -2 & 1 \end{array}\right]$

$D = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & -2 \end{vmatrix} = 10$ $D_x = \begin{vmatrix} 3 & -1 & 1 \\ 4 & -1 & -1 \\ 1 & 2 & -2 \end{vmatrix} = 14$

$D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 4 & -1 \\ 1 & 1 & -2 \end{vmatrix} = -14$ $D_z = \begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = -12$

$x = \frac{D_x}{D} = \frac{14}{10} = 1.4$ $y = \frac{D_y}{D} = \frac{-14}{10} = -1.4$

$z = \frac{D_z}{D} = \frac{-12}{10} = -1.2$

$\boxed{(1.4, -1.4, -1.2) \text{ or } (7/5, -7/5, -6/5)}$

20. $\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 2+(-3) & 1+4 \\ 4+(-2) & -3+7 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 5 \\ 2 & 4 \end{bmatrix}}$

21. $3\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix} - 2\begin{bmatrix} -3 & 4 \\ -2 & 7 \end{bmatrix} =$

$\begin{bmatrix} 6 & 3 \\ 12 & -9 \end{bmatrix} - \begin{bmatrix} -6 & 8 \\ -4 & 14 \end{bmatrix} = \begin{bmatrix} 6-(-6) & 3-8 \\ 12-(-4) & -9-14 \end{bmatrix} = \boxed{\begin{bmatrix} 12 & -5 \\ 16 & -23 \end{bmatrix}}$

22. $\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}} = \begin{bmatrix} (1)(1)+(2)(-1)+3(2) & (1)(2)+(2)(0)+3(4) \\ (0)(1)+(-1)(-1)+4(2) & (0)(2)+(-1)(0)+4(4) \end{bmatrix}$

$= \boxed{\begin{bmatrix} 5 & 14 \\ 9 & 16 \end{bmatrix}}$

product is 2×2

23. $\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right] \xrightarrow{R_1 = r_1 - r_2} \left[\begin{array}{cc|cc} 1 & -5 & 1 & -1 \\ 2 & 4 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_2 = -2r_1 + r_2} \left[\begin{array}{cc|cc} 1 & -5 & 1 & -1 \\ 0 & 14 & -2 & 3 \end{array}\right] \xrightarrow{R_2 = r_2/14} \left[\begin{array}{cc|cc} 1 & -5 & 1 & -1 \\ 0 & 1 & -1/7 & 3/14 \end{array}\right]$

$\xrightarrow{R_1 = 5r_2 + r_1} \left[\begin{array}{cc|cc} 1 & 0 & 2/7 & 1/14 \\ 0 & 1 & -1/7 & 3/14 \end{array}\right]$ $\boxed{A^{-1} = \begin{bmatrix} 2/7 & 1/14 \\ -1/7 & 3/14 \end{bmatrix}}$

24. $\begin{cases} x + 2y + 3z = 2 \\ x + y + z = -3 \\ -x + y + 2z = \end{cases}$ $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$

$A \quad X = B$

$X = A^{-1} \quad B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -3 & 5 & 2 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ -13 \\ 9 \end{bmatrix}$ $\boxed{(1, -13, 9)}$

25. $\dfrac{x}{x^2+7x+12} = \dfrac{x}{(x+3)(x+4)} = \dfrac{A}{x+3} + \dfrac{B}{x+4}$

$x = A(x+4) + B(x+3)$

$x = Ax + 4A + Bx + 3B$

$x: \ 1 = A + B$

$\text{cons}: 0 = 4A + 3B$

$-3(A + B = 1)$

$4A + 3B = 0$

$-3A - 3B = -3$

$A = -3$

$-3 + B = 1 \quad B = 4$

$\boxed{\dfrac{-3}{x+3} + \dfrac{4}{x+4}}$

26. $\dfrac{3x^2-2x+4}{(x+1)(x-1)^2} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

$3x^2-2x+4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$

$3x^2-2x+4 = A(x^2-2x+1) + B(x^2-1) + Cx + C$

$3x^2-2x+4 = Ax^2-2Ax+A+Bx^2-B+Cx+C$

x^2: $3 = A+B$

x: $-2 = -2A+C$

Const: $4 = A-B+C$

$$\text{rref}\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ -2 & 0 & 1 & -2 \\ 1 & -1 & 1 & 4 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 9/4 \\ 0 & 1 & 0 & 3/4 \\ 0 & 0 & 1 & 5/2 \end{array}\right]$$

$A = 9/4$
$B = 3/4$
$C = 5/2$

$$\boxed{\frac{9/4}{x+1} + \frac{3/4}{x-1} + \frac{5/2}{(x-1)^2}}$$

27. $\dfrac{2x+1}{(x+1)(x^2+x+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+x+1}$

$2x+1 = A(x^2+x+1) + (Bx+C)(x+1)$

$2x+1 = Ax^2+Ax+A+Bx^2+Bx+Cx+C$

x^2: $0 = A+B$

x: $2 = A+B+C$

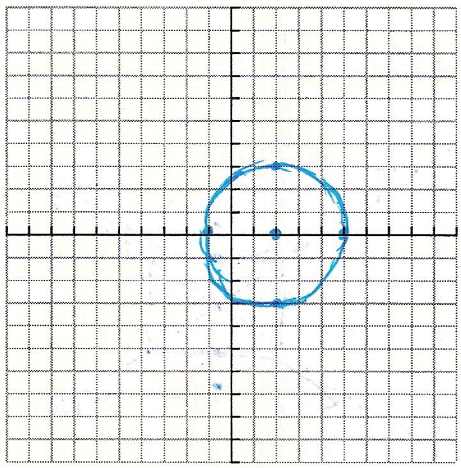
const: $1 = A+C$

$$\text{rref}\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$A = -1$
$B = 1$
$C = 2$

$$\boxed{\frac{-1}{x+1} + \frac{x+2}{x^2+x+1}}$$

1.



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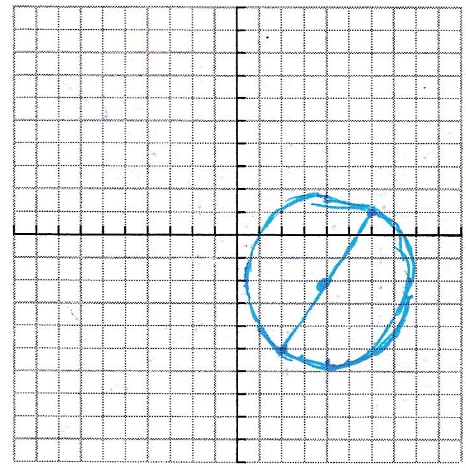

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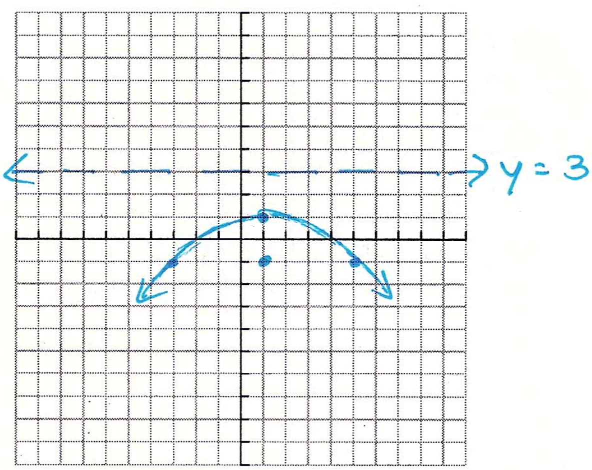

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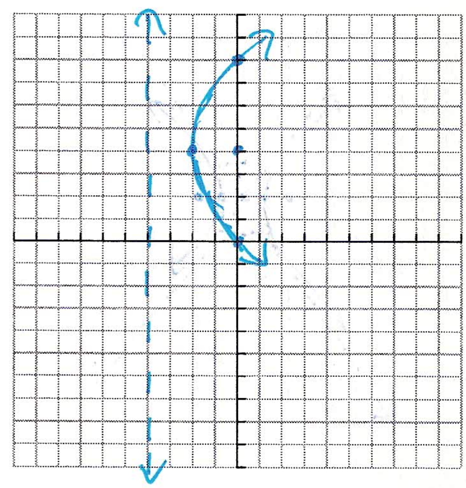

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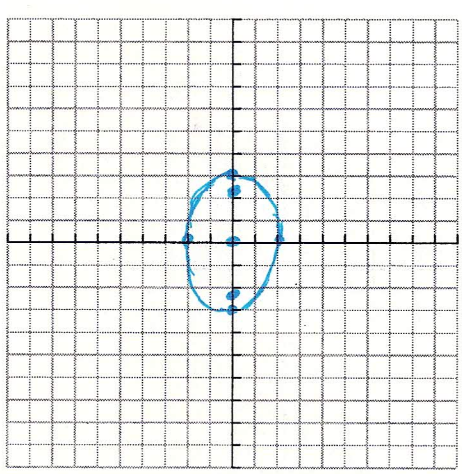

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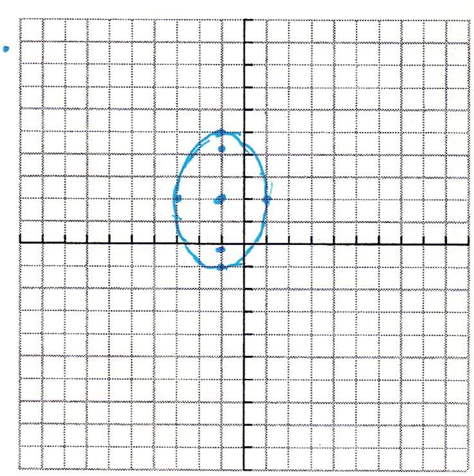

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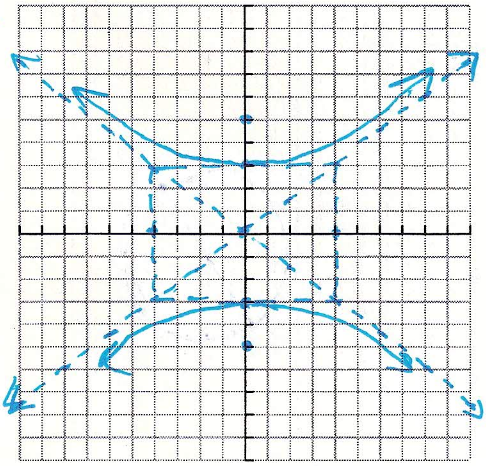

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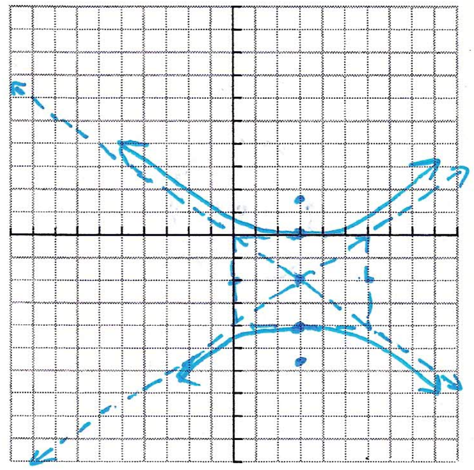

11.